Resolution improvement of data-independent beamformers

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ABSTRACT

Beamforming microphone antenna has been widely used to locate sound sources and build acoustic model. The particularities of this technique are the use of a relatively few number of microphones and the simple signal processing. So, this technique is fast and easy to be carried out. But its main inconvenience is the poor resolution at low frequencies. Other approaches using experimental data have been proposed to increase the resolution, for example, minimum variance method, and high-resolution methods. However these methods are often delicate to put in use in acoustic engineering or they are often reproached for lack of robustness. Whereas, the most used methods nowadays are those by which the determination of the steering vector is data-independent. Among those methods we consider the conventional beamforming processing (Bartlet processor), the techniques of reduction of diagonal load of the microphone cross-spectral matrix, the method of using a Statistically Optimal Array Processing (SOAP) completed by the techniques of post-processing deconvolution.

1 INTRODUCTION

The beamforming techniques have now been widely used to analyse fixed and moving sound sources. Localizing a noise source and analyzing the characteristics of its radiation can provide a global evaluation of industrial problem, a better understood of the physical mechanisms of noise generation and an aid in constructing noise generation models. Array processing has long been used as spatial filter in multiple fields, such as radar, sonar, communications, geophysics, astronomy, biomedical imaging and more recently in engineering acoustics, for the purpose of obtaining accurate spatial information about sound sources [1-7].

Different aspects of microphone array processing have been discussed in the literature [8]. One of the first techniques investigated to locate acoustic sources is the conventional beamforming method, or delay-and-sum (DAS) beamformer. It has the advantage of being robust and easy to implement. High side lobes and varying main lobe width as a function of frequency are the inherent problems of the delay-and-sum beamforming method. One approach to solve this problem is to use weighting algorithms with the delay-and-sum method [9, 10, 5]. An adaptive beamformer, such as Capon beamformer (1969), and high resolution beamformer, has better resolution and much better interference rejection capability than data-independent beamformers, provided that the array steering vector corresponding to the source radiation is accurately known [11,13,12]. However, there are many factors in practice that
can degrade the performance of the adaptive beamformer, like errors due to inaccurate knowledge of transfer function between sources and microphones and array cross-spectral matrix estimation errors due to self noise and statistical uncertainties. The applications in the acoustic fields undoubtedly pose much more problems than those in other fields, because the sizes of the sources, obstacles and propagation areas are in the same order of magnitude as the wavelength. Owing to the previous problems, the performance of the adaptive and high resolution beamformer may become worse than that of the standard data-independent beamformers, such as the delay-and-sum (DAS) beamformer. Consequently these beamformers have not been used very much in practice despite their potential advantages and much work done to improve their robustness [14,15].

An alternative solution between the delay-and-sum (DAS) conventional beamforming method and the adaptive beamforming methods is for the goal to optimize the steering vector for the scanning source region without using the measured cross-spectral matrix. The SOAP method (Statistically Optimized Array Processor) uses statistical optimization of the steering vector to improve the beampattern resolution [16].

The objective of this paper is to compare the above mentioned beamforming methods which do not use the measurement data to build the steering vector. Complementary techniques like the removal of the diagonals of the cross-spectral matrix and the post-processing of deconvolution are proposed to increase the resolution [18,5]. The performances of the methods are evaluated by the consideration of their robustness.

## 2 DATA-INDEPENDENT BEAMFORMING TECHNIQUES

### 2.1 Delay-and-sum (DAS) conventional beamforming method

At a frequency \( f = \omega/2\pi \), an estimation of the amplitudes of the sources can be made from the vector of the complex pressures measured by an antenna consisting of \( M \) microphones. The amplitude of the source at the focal point \( i \) can be written formally by

\[
S_i = w_i^H p,
\]

where \( w_i \) is the steering vector associated the focal point \( i \) (\( H \) denotes the Hermitian transpose). The steering vector is based on the estimate of the vector \( h_i \), the transfer functions at point de focalisation explored to the antenna of microphones. For example, for a point source in a free space, each element of \( h_i \) is \( \exp(-jkR_{im})/R_{im} \), where \( k = 2\pi f/c \) and \( R_{im} \) is the distance between the source \( i \) and the microphone \( m \). In the case of the Bartlet processor (maximum likelihood), \( u_i = \text{diag}(a)h_i \), where \( a \) is the weight vector corresponding to a spatial window (Hanning, Kaiser-Bessel, Blackman, etc.) used to reduce the side lobes. \( w_i \), which satisfies to the normalisation constraint \( w_i^H h_i = 1 \), is expressed as

\[
w_i = u_i/(u_i^H h_i).
\]

The square amplitude of the distribution of source can be written by

\[
|S_i|^2 = (w_i^H p)(w_i^H p)^H = w_i^H p p^H w_i = w_i^H G_{pp} w_i,
\]

where \( G_{pp} \) is cross-spectral matrix of pressures. It is this form which is used in practice because it allows one to consider partially coherent sound fields [19] which cannot be represented by a simple vector of pressure.

### 2.2 Statistically Optimal Array Processing (SOAP) method

The SOAP method was proposed as an alternative method between the conventional and the adaptive beamforming methods [16]. Optimization is not made by using the cross-spectral
matrix measured but by using a cross-spectral matrix model obtained by considering that on each grid point of the scanning source plane is located a point source having a unity amplitude. This method is based on the same concepts as those used by Hald [21] for obtaining an optimization reconstruction algorithm of patch acoustic holography (SONAH). Considering the matrix \( W = [w_1, \ldots, w_r, \ldots, w_L] \) of the steering vectors of all the focal points in the scanning plane, the column vector \( s \) of the \( L \) amplitude source points of elements \( S_i \), defined by equation (1) is given by

\[
s = W^H p.
\]

For a single source with unity amplitude located at \( r_i \), the vector of pressures can be expressed by \( h_i \) and the source vector is given by

\[
s_i = W^H h_i.
\]

In the ideal case, all elements of this vector should be equal to zero except for the element \( l \) with a value equal to 1. Actually, the vector with the square of the elements of the vector \( s_i \) represents the power pattern of the antenna, defining its response when the beamformer scans all points of the plan with a source located at \( r_i \). It is possible to synthesize this information for sources occupying all the points of the scanning plane in the form of the expression,

\[
S = W^H H,
\]

where \( S = [s_1, \ldots, s_r, \ldots, s_L] \) and \( H = [h_1, \ldots, h_r, \ldots, h_L] \). It is expected that the matrix \( W \) will be such that it results \( S \) in an identity matrix with ones on the diagonal and zeros elsewhere. The problem is thus to find the matrix \( O \) which verifies the following equation,

\[
I = O H.
\]

The above equation has the solution only in the Least Mean Square (LMS) sense. The examination of Eq. (7) shows that it corresponds to the regularized pseudo-inverse of the matrix \( H \),

\[
O = H^+ = H^H (H^H + \beta I)^{-1}, \quad \text{for} \quad \beta > 0.
\]

Thus, in the case of the method SOAP, the steering vector is written from \( u_j = \alpha h_j \), where \( \alpha = [HH^H + \beta I]^{-1} \) that is of a Hermitian symmetry \( (\alpha^H = \alpha) \). One can obtain,

\[
w_i = (\alpha h_i) / (h_i^H \alpha h_i).
\]

The regularization parameter \( \beta \) corresponds to the variance of the self noise on the microphones of the antenna. It can also be determined, in practice, by the generalized cross validation technique [17]. This method determines the value of \( \beta \) using a minimization procedure which does not require knowledge of the noise variance. In this paper, \( \beta \) is expressed by a signal-to-noise ratio in dB as the following form

\[
\beta = 10 \frac{\text{SNR}}{10} \max \{ \text{diag}(G_{pp}) \}.
\]

The increase in the value of the regularization parameter will allow the increase in the robustness of the estimate in the presence of noise, with the detriment of the resolution of the beamformer.

### 2.3 Cross-spectral formulation with exclusion of auto-spectra

The auto-spectra in the diagonal of the cross-spectral matrix of pressures are often contaminated by self-noise on microphone channels (electronic noise, quantification noise,
aerodynamic noise, etc.) whereas the cross-spectral components are not sensitive to the above mentioned noises because these noises are independent of each other. This observation leads to the proposition to delete the diagonal elements of the cross-matrix which does not contain the information of phase among microphones [5,18,20]. The estimator then becomes,

\[ |S_i|^2 = w_i^H G_{pp} w_i - w_i^H \text{diag}(G_{pp}) w_i, \]  

which subjects to the constraint of normalisation \( w_i^H h_i h_i^H w_i - \text{tr}(w_i h_i h_i^H w_i)^2 = 1. \)

![Figure 1 – Example of localisation of 3 independent sources (yellow square marks) for Conventional Beamforming (CBF) without and with diagonal exclusion of autospectra and SOAP processing with a regularization parameter of 15 dB. Top : 800 Hz, middle : 1600 Hz, bottom : 3200 Hz. Map dynamic range : 15 dB.](image)

The comparison of the three approaches of data-independent beamforming is shown in Fig. 1 with an unusual dynamic range of 15 dB in order to well show the side lobes. The results are obtained by the simulation of three point sources located at 1.5 m from a small 8-arm antenna (aperture: 0.96 m, 49 microphones with spacing 0.08 m). The noise spectral density \( G_{NN} \) is added to the diagonal elements of the cross-spectral matrix of pressures. The signal-to-noise ratio is 20 dB. It is supposed here that the three point sources are independent of each other. The exclusion of the diagonal elements of the matrix results in the reduction of the interference side lobes, especially at high frequencies, whereas SOAP method improves the resolution at low frequencies, in spite of a significant parameter of regularization (15 dB).
Figure 2 shows the same configurations as those in Fig. 1 at 3200 Hz except for the case of coherent sources. It is noticed that the interferences among sources do not have sensitive effect on the results at frequency less than 2000 Hz. It is also shown that an increase in the amplitude of side lobes due to the interferences, and that the exclusion of the diagonal elements of the cross-spectral matrix maintains the level lower than that of the other methods.

Figure 2 – Example of localisation of 3 coherent sources (yellow square) for Conventional Beamforming (CBF) without and with diagonal exclusion of autospectra and SOAP processing with a regularization parameter of 15 dB at 3200 Hz. Map dynamic range: 15 dB.

3 POST PROCESSING DECONVOLUTION TECHNIQUES

The techniques of deconvolution of images have been used for a long time in astrophysics and some have been adopted in acoustic imaging like the method CLEAN [22]. Certain methods such as deconvolution approach [18] use an inverse technique to restore the sound power levels of interest from the output of conventional beamforming. The numerical simulations and experimental applications presented in Ref. [18] have shown that the deconvolution approach for the mapping of acoustic sources (DAMAS) is appropriate for improving the localisation of the acoustic sources. Other algorithms are, however possible to carried out for this operation and they are briefly examined in this article.

By considering the response in power of the antenna for a source \( n \): \( Y_n = w_n^H G_{pp} w_n \).

The model of the cross-spectral matrix of the pressures for a single source of quadratic amplitude \( X_n \) is then written as \( \tilde{G}_n = X_n h_n h_n^H \). With the assumption of uncorrelated sources the model of the matrix of the pressures for all the sources is \( \tilde{G} = \sum_n \tilde{G}_n = \sum_n X_n h_n h_n^H \). In this case the response becomes

\[
\tilde{Y}_n = w_n^H \tilde{G} w_n = w_n^H \left( \sum_i X_i h_i h_i^H \right) w_n = \sum_i w_n^H h_i h_i^H w_n X_i = \sum_i A_{ni} X_n, \tag{12}
\]

where \( A_{ni} = w_n^H h_i h_i^H w_n \). The matrix form of Eq. (12) is expressed by \( Y = AX \), where \( Y \) is the response in power with elements \( \tilde{Y}_n = |S|^2 \) calculated from the output of beamforming processing. \( X \) is the required vector of the quadratic amplitude of the sources and \( A \) the model matrix. The solution is written in the form

\[
X = A^{-1} Y, \tag{13}
\]

and is obtained by an iterative inversion algorithm. The \( n^{th} \) row of the matrix \( A \) is the discrete form of the diffusion function of the point \( n \) on the whole image. This diffusion function named point spread function (PSF) has well known characteristics for beamforming processing.
3.1 DAMAS (Gauss-Seidel) algorithm

The DAMAS method [18] uses the technique of Gauss-Seidel to carry out the deconvolution. By writing a row in the following form
\[ Y_n = A_{n1} X_1 + A_{n2} X_2 + \cdots + A_{nm} X_m + \cdots + A_{nN} X_N, \]  
(14)
it is possible to express \( X_n \) in the form
\[ X_n = \frac{1}{A_{nm}} \left( Y_n - \sum_{k=1}^{n-1} A_{nk} X_k - \sum_{k=n+1}^{N} A_{nk} X_k \right). \]  
(15)
The algorithm of Gauss-Seidel for the iteration \( i \) is
\[ X_n^{(i)} = \frac{1}{A_{nm}} \left( Y_n - \sum_{k=1}^{n-1} A_{nk} X_k^{(i)} - \sum_{k=n+1}^{N} A_{nk} X_k^{(i-1)} \right). \]  
(16)
The initial values \( X_n^{(0)} \) are chosen to be equal to \( Y_n \). The convergence is made under positive constraint, i.e. if a value of \( X_n \) is negative, it is put to 0. In reference [18] it is recommended to carry out the algorithm successively in ascending order (from \( n = 1 \) to \( n = N \)) then in descending order (from \( n = N \) to \( n = 1 \)) but the test did not show gain in the speed of convergence. However, it is appreciably improved by choosing the order of the decreasing amplitude of sources.

3.2 Richardson-Lucy's algorithm

The Richardson-Lucy algorithm (Lucy 1974) is an iterative procedure for recovering a latent image that has been blurred by a known point spread function. It is proposed by Papamoschou and Dadvar [23] as a method more powerful than DAMAS for the deconvolution of the information at output of the beamforming processing. The iteration is written by
\[ X_n^{(i)} = X_n^{(i-1)} \frac{\sum_m A_{mn} Y_m^{(i-1)}}{\sum_m A_{mn}} \text{ with } Y_m^{(i-1)} = \sum_k A_{mk} X_k^{(i-1)}. \]  
(17)
It has been proved that if this iteration converges, it converges to the maximum likelihood solution. Richardson-Lucy algorithms lead to satisfactory reconstructions and the results are very stable, especially in situations with bad signal-to-noise ratio.

3.3 Jansson-Van Cittert's algorithm

The iteration in matrix form is built by the following way
\[ X^{(i)} = X^{(i-1)} + \alpha (Y - X^{(i-1)}) = X^{(i-1)} + \alpha (Y - AX^{(i-1)}). \]  
(18)
\( \alpha \) is a vector of the relaxation coefficients. This iteration was first defined by Van Cittert (with \( \alpha = 1 \)). This technique, firstly developed for application in spectroscopy, can converge with relatively a few iterations but diverges in the presence of noise. Dudgeon and Mersereau [8] gave a convergence condition as \( |1 - \alpha A_n| < 1 \). Jansson proposed to determine the relaxation coefficient by considering the constraints imposed on the solution \( X_{\text{min}} < X < X_{\text{max}} \)
\[ \alpha = c \left[ 1 - 2 \left( \frac{X}{X_{\text{max}}} + X_{\text{min}} \right) \right]. \]  
(19)
For our problem it is possible to choose \( X_{\text{min}} = 0 \), as Conchello and Hansen [24] did (where \( c = 0.005 \) and \( q = 1 \)), and in imposing the constraint of the positive values (\( X_n^{(i-1)} \geq 0 \)).
Figure 3 – Deconvolution post-processing techniques (from top to bottom: DAMAS ascendant order, DAMAS ascendant and descendent order, DAMAS decrease amplitude order, Richardson-Lucy and Jansson-Van Cittert) from the output data of the Conventional Beamformer (CBF) at 1000 Hz. Map dynamic range: 15 dB. The first column is a relative convergence criterion versus the number of iterations.
Fig 3 shows the comparison of the three deconvolution techniques for the DAS conventional beamforming (CBF) at 1000 Hz and for the same 3-source configuration as that for Fig. 1. For DAMAS, the speed of convergence depends on the order chosen for the iterations. The fastest is the order of the decreasing amplitude of the sources which is retained for the following comparison. Richardson-Lucy’s method has tendency to smooth the results, whereas Jansson-Van Cittert’s method converges in general very fast, but towards to a result that presents important residual lobes. With the last method the determination of the parameter of relaxation is sometimes delicate and can lead the algorithm of diverging.

Figure 4 – Deconvolution post-processing using DAMAS and Richardson-Lucy algorithms for the configuration of Figure 1 from the output data of the Conventional Beamformer (CBF) and the SOAP beamformer with a regularization parameter of 15 dB. Top: 800 Hz, middle: 1600 Hz, bottom: 3200 Hz. Map dynamic range: 15 dB.

Sometimes when the sources are close to each other, the iterative algorithms do not work to separate them, as it is shown in Fig. 4 for the CBF with the technique of Richardson-Lucy. SOAP method, in this case, demonstrates real advantage over other methods. To verify the robustness of these techniques, the model of cross-spectral matrix simulated has been disturbed by a Gaussian random signal (this random component disturbs also the phase of the cross-spectra but preserves the Hermitian symmetry of the matrix). For a standard deviation of $-6$ dB, expressed with respect to the amplitude of the microphone signals, practically no deterioration of the results are perceptible. Drop-out starts to appear only when this standard deviation approaches $0$ dB.
4 CONCLUSIONS

The main inconvenience of the conventional data-independent beamforming technique is the increasing width of the main lobe with the decreasing frequencies. This lack of resolving capability limits dramatically the use of this technique to locate noise sources in engineering applications where the frequency band of interest is wider than that the antenna allows generally.

The use of the SOAP method makes it possible to improve the resolution at low frequencies, as much better than the measurement noise or the disturbance of the model remain weak. The technique of exclusion of the diagonal of the cross-spectral matrix of the measured pressures does not, in fact, improve the resolution but contributes to reduce the side lobes, more particularly those due to the interferences in the higher part of the frequency range of the antenna.

The deconvolution techniques recently proposed seem to be much of interest like techniques of post-processing. The DAMAS method (Gauss-Seidel) presents a good resolution whereas the algorithm of Richardson-Lucy converges towards a smooth but also more stable solution. Some difficulties may occur in the estimation process if information concerning the acoustic sources of interest is masked by a too wide main lobe at low frequencies. In this case, SOAP method gives advantages.

It is noticed that these techniques are robust by considering the noises and uncertainties on the propagation model. Examples of simulations show that the random component disturbing the model should have the same order of magnitude as that of the auto-spectra of the measured microphone signals to reach the limits of using these techniques.

5 REFERENCES


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