IRROTATIONAL ACOUSTIC INTENSITY:  
A NEW METHOD FOR LOCATION OF SOUND SOURCES

Jean-Claude Pascal¹, Jing-Fang Li²

¹ Institut d’Acoustique et de Mécanique (IAM) et  
Ecole Nationale Supérieure d’Ingénieurs du Mans (EMSIM)  
Université du Maine, rue Aristote, 72085 Le Mans Cedex 09, France  
² ORION, 51 rue d’Alger, 72000 Le Mans, France

ABSTRACT

The new conception of acoustic intensity is developed in this paper. Since it is  
derived from the calculation of the gradient of the scalar potential of the active intensity  
that characterizes a sound field without vorticity; we call it the irrotational acoustic  
intensity. The acoustic holography technique allows 3-dimensional sound fields in space  
to be obtained using the complex pressure or intensity on the measuring plate near sources.  
It is possible to calculate the scalar potential from the 3-dimensional sound fields, and  
the irrotational acoustic intensity can be derived from this potential. Analyses are made  
to show the application of the irrotational intensity in the location of sound sources.  
The technique developed in this paper is compared with the existing technique such as  
the standard acoustic intensity and the supersonic acoustic intensity for identification  
of region on a planar vibrating source. Useful information about location and identification  
of sound sources by the irrotational intensity is given.

1. INTRODUCTION

Identification and location of sound source are very important in the noise control  
engineering. Over the past two decades, the technique of acoustic intensity measurements  
has been widely used for location and analysis of sound sources on structures which radiate
to the far field. A lot of theoretical and experimental work in this field can be found in edited books, journal publications and congresses, for example Refs. [1, 2, 3]. Structural intensity, which describes the power transferred by elastic waves through mechanical structures, helps us to determine how the energy is injected by mechanical excitations and what transfer paths it takes to transfer vibrations to other connected structures [2]. The divergence of the structural intensity allows mechanical excitation or damping on the structures to be located and identified [3, 4].

Acoustic holography allows is also recognized as a powerful tool to characterize sound sources and to estimate the structural energy flow and far-field directivity. This technique has been developed from laboratory NAH [5] to more practically-used methods such as BAHIM [6], STSF [7, 8].

It is well known the vortex in the near-field of a structure makes it difficult for the noise control engineer to locate, from near-field measurements, the acoustic sources which radiate to the far field, since the latter arises from imperfect cancellation of adjacent regions on the measurement surface. In order to solve this problem, a conception of supersonic acoustic intensity is introduced first for cylindrical structure in 1995 and then developed in planar plates in 1998 [9, 10]. The supersonic intensity is composed only of wave components which radiate to the far field, with the non-radiating components eliminated.

In this paper, we introduce a new concept: an irrotational sound intensity which is defined as the gradient of the active sound intensity potential. As it is a non-vorticity, it presents a field without curl path patterns among sources. First the vector of the irrotational sound intensity is defined from the vector expressions for a sound field. The use of this conception will be demonstrated in the second section. Comparisons between the irrotational sound intensity and standard acoustic intensity, supersonic intensity are made to show source location methods.

2. DEFINITION OF IRROTATIONAL SOUND INTENSITY

The purpose of this section is to define the irrotational sound intensity. First the nature of sound intensity field is recalled. The irrotational sound intensity is then introduced.

2.1 Vector representation of a sound intensity field.

The divergence and curl of a vector field are two important variables to study the characteristics of a vector field such as the sound intensity field. The divergence of a vector is a scalar that is related to a ‘source’ or a ‘sink’ of the vector field. The field is solenoidal in a region in which the divergence is zero. A potential vector can be introduced to describe the solenoidal sound field. The curl of a vector field is used to analyze the interferences resulting in the interactions among sources which generated sound fields. If the curl is zero, the vector field is conservative and a scalar potential is used to describe the field. A sound intensity field is, in general, neither a purely conservative nor a purely solenoidal field. The divergence and the curl of the active sound intensity \( I \) are written in the following forms:

\[
\nabla \cdot I = \omega(r_0), \quad \nabla \times I = \Omega,
\]

(1)
where \( w(\mathbf{r}_0) \) is the volume density of power, \( \mathbf{r}_0 \) is the coordinates of ‘sources’ or ‘sinks’, \( \mathbf{\Omega} \) is the vector potential of the active intensity. We introduce two components \( \mathbf{I}_\phi \) and \( \mathbf{I}_c \), the active intensity can be written as the superposition of the two components:

\[
\mathbf{I} = \mathbf{I}_\phi + \mathbf{I}_c,
\]

where \( \mathbf{I}_\phi \) is of purely conservative so that it is defined by the following equations,

\[
\nabla \cdot \mathbf{I}_\phi = w(\mathbf{r}_0), \quad \nabla \times \mathbf{I}_\phi = 0, \tag{3a}
\]

\( \mathbf{I}_c \) is of purely solenoidal, its divergence and curl are expressed by

\[
\nabla \cdot \mathbf{I}_c = 0, \quad \nabla \times \mathbf{I}_c = \mathbf{\Omega}. \tag{3b}
\]

A scalar potential \( \phi \) can be defined in the vector field \( \mathbf{I}_\phi \). The relation between \( \mathbf{I}_\phi \) and \( \phi \) is given as follows:

\[
\mathbf{I}_\phi = -\nabla \phi, \tag{4}
\]

Substituting Eq. (3a) into Eq. (4) yields the Poisson equation

\[
\nabla^2 \phi = -w(\mathbf{r}_0). \tag{5}
\]

The solution of the Poisson Equation is

\[
\phi(\mathbf{r}) = \iiint_V \frac{w(\mathbf{r}_0)}{||\mathbf{r} - \mathbf{r}_0||^3} \, d\mathbf{r}. \tag{6}
\]

Eq. (6) shows that the scalar potential of the irrotational sound intensity is the function of the distribution of ‘sources’ or ‘sinks’, which depends on the interactions among sources mutual radiation impedance. In no ‘sources’ or ‘sinks’ regions, Eq. (5) becomes the Laplace equation

\[
\nabla^2 \phi = 0. \tag{7}
\]

Therefore the scalar potential allows us to obtain the information about the sources such as the acoustic power, far-field directivity. Whereas the component \( \mathbf{I}_c \) represents the circulation of the sound power in near-field of structures and the interference configurations of sound fields. This interference makes it difficult to locate the acoustic sources and analyze the radiation phenomena in the near-field regions. To solve this problem, Williams [9] introduced supersonic intensity which is obtained using the Fourier transform to eliminate evanescent waves, leaving only the far-field radiating components. Since the negative intensity regions of a vibrator are removed, sources of radiation are readily located on the surface of the vibrator. In this work we propose a sound intensity which is composed only of the first term of Eq. (2) with the second term eliminated. As it is related to the scalar potential due to irrotational vector field (\( \mathbf{\Omega} \neq 0 \)), we call it an irrotational sound intensity. The calculation method of the irrotational sound intensity is given in the following subsection.

2.2 Irrotational sound intensity. If the two components in Eq. (2) can be separated, the irrotational sound intensity can be easily calculated. However the separation of the
two vector fields is, in general, impossible except for a few of simple sound fields like two plane waves propagating in perpendicular directions \[ \text{[11, 12]} \]. Eq. (4) shows that the irrotational sound intensity is the gradient of the scalar potential. The solution of Equation (4) can also be obtained using the Fourier transform. Applying the Fourier transform to Eq. (4) yields:

\[
\Phi(K_x, K_y, K_z) = -j K \cdot I_\phi(K_x, K_y, K_z) \quad \text{K}^2
\]

where \( F \{ \nabla \phi(x, y, z) \} = -j K \cdot \Phi(K_x, K_y, K_z) \) is used, with \( K = (K_x i + K_y j + K_z k) \) (time dependent term \( e^{j\omega t} \) is used in this paper).

Substituting \( \Phi = I - I_C \) and using \( I_C = \nabla \times C \), the irrotational sound intensity in Eq. (8) can be replaced by the total sound intensity \( I \) in the 3D wave number domain,

\[
\Phi(K_x, K_y, K_z) = -j K \cdot I(K_x, K_y, K_z) \quad \text{K}^2
\]

Equation (9) shows that a three dimensional total active intensity \( I(K) \) should be known in order to calculate the scalar potential. One needs to measure the 3D acoustic intensity vector near sources and then perform the 3D Fourier transform. But it is very time consuming and often prohibitive. The acoustic holography provides a powerful method by which the sound field in a full space can be reconstructed using in-plane measurement data. Once \( \Phi(K_x, K_y, K_z) \) is obtained the irrotational sound intensity \( I_\phi(K_x, K_y, K_z) \) can be calculated using

\[
I_\phi(K_x, K_y, K_z) = -j K \cdot \Phi(K_x, K_y, K_z).
\]

The irrotational sound intensity can be obtained by the use of the inverse Fourier transform,

\[
I_\phi(x, y, z) = F^{-1} \{ I_\phi(K_x, K_y, K_z) \}.
\]

3. NUMERICAL EXAMPLE OF THE IRROTATIONAL SOUND INTENSITY

To demonstrate the concept of irrotational sound intensity and its ability to locate the regions on a structure, we consider a simply supported plate vibrating unforced in a nature mode of which normal vibrating velocity is computed using modal analysis:

\[
v(x, y) = \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y}.
\]

where \((x, y)\) is a point coordinate, \(L_x\) and \(L_y\) are the width and length of the rectangular plate, \((m, n)\) is the \(m\)-th and \(n\)-th vibration mode of the plate.

The technique of Fourier transform is used for numerical calculation. The processing algorithm in wave number domain to calculate the irrotational sound intensity from a knowledge of a mode vibration \(v(x, y)\) is implemented as follows:

(i) Calculate the discrete Fourier transform of \(v(x, y)\) and call it \(V(K_x, K_y)\).

(ii) Calculate the pressure spectrum on the plane parallel to the plate with distances \([z_1, z_2, \ldots, z_{N_z}]\) using the averaged Green function: \(P(K_x, K_y, z) = V(K_x, K_y)G(K_x, K_y, z)\) [13].
(ii) Perform Fourier transform of \( P(K_x, K_y, K_z) \) in \( z \)-direction to obtain the volume pressure \( P(K_x, K_y, K_z) \) and velocity \( U(K_x, K_y, K_z) \).

(iv) Calculate three-dimensional inverse Fourier transform respectively on the pressure and velocity to get volume space \( p(x, y, z) \) and \( u(x, y, z) \) which are used for calculation of the space volume active sound intensity vector: \( I(x, y, z) = 1/2 \Re\{p(x, y, z)u(x, y, z)^*\} \).

(v) Calculate Fourier transform on the volume active sound intensity \( I(K_x, K_y, K_z) \). Then calculate the volume scalar potential \( \Phi(K_x, K_y, K_z) \) using Eq. (9).

(vi) Calculate the irrotational sound intensity \( I_0(K_x, K_y, K_z) \) in wave number domain using Eq. (10). The irrotational sound intensity \( I_0(x, y, z) \) is obtained by calculation of inverse Fourier transform given by Eq. (11).

---

Fig 1 - Normal vibration velocity (top left), standard sound intensity (top right), irrotational sound intensity (bottom left) and supersonic sound intensity (bottom right) for a mode \((11, 9)\) of a baffled plate with \( L_x = L_y = L = 2m \) and \( f = 500 \) Hz. The baffle is shown surrounding the plate.

For an example, the mode \((11, 9)\) and \( L_x = L_y = L = 2m \) are used in the calculation. The frequency \( f = 500 \) Hz which gives \( \lambda = 0.66 \) m and \( kL = 15.95 \). At this frequency the plate is excited below critical frequency, since \( \lambda \) is greater than both \( \lambda_m = 2L/m = 0.36 \) m and \( \lambda_n = 2L/n = 0.44 \) m so that \((11, 9)\) is a corner mode
The aperture of the plane for calculating is a double size of the plate. The lattice spacing is 3.06 m. Fig. 1 shows the normal vibration velocity on the plate surface for the mode (11,9), the standard active intensity $I(x, y) = 1/2 \text{Re}\{p(x, y)v(x, y)\}$. The supersonic sound intensity for the normal mode (11,9) is also shown in Fig. 1 for comparison. The negative and positive normal intensity is presented by the gray-scale plot in dB. The gray scale level is ± 10 dB. It is noted that the irrotational sound intensity is very different from the standard sound intensity. The standard sound intensity shows the vortex effect of the rotational part of the sound intensity. Therefore it is hard to tell the far-field sound regions from the standard intensity. The irrotational sound intensity, however, shows the dominant positive intensity regions at the four corners of the plate which radiate to the far field. These results have a good agreement with the theory of mode classification [14]. Compared with the supersonic sound intensity, the irrotational sound intensity gives the same resolution about the real sources though averaging the adjacent negative and positive intensity. It is indicated that the spatial resolution of the supersonic-intensity method is limited to $\lambda/2$.

To demonstrate the spatial resolution for location of source by the irrotational sound intensity, we consider a normal mode (4,3). In order to have the same structural wavelengths as those in the previous example, the plate dimension should be $L_x = 4L/11$, $L_y = 3L/9$. The frequency of calculation is still $f = 200$ Hz. In Fig. 2 is shown the comparison of the three different forms of acoustic intensity. It is noted that there are positive and negative irrotational sound intensity, the positive intensity on the four corners of the plate is dominant throughout. It shows that the corners are the four dominant sources on the plate which radiate the sound power to far-field. The supersonic sound intensity gives the similar information but its resolution is limited to $\lambda/2$.

![Fig. 2 - Standard sound intensity (left), irrotational sound intensity (middle) and supersonic sound intensity (right) for a mode (4,3) of a baffled plate with $L_x = 4L/11$ and $L_y = 3L/9$ ($L=2$ m). The frequency of calculation is 500 Hz. The structural wavelengths are the same as the results in Fig. 1. The contour line delimits the -3 dB bandwidth.](image)
The third example is a source model composed of three coherent monopoles in line. The monopoles have the same amplitude and their initial phases are respectively 0°, 180° and 0°. They produce pressure and particle velocity which were computed using source model software [15]. Then the particle velocity was used to calculate the sound intensity using the Fourier transform. Fig. 3 shows the sancard sound intensity, the irrotational intensity as well as the supersonic intensity. From the irrotational intensity, it is shown that the two in-phase monopoles modified the radiator impedance of the monopole in middle and made its energy flow being negligible. It is these two monopoles who mainly contribute to the radiation in the far-field.

4. CONCLUSION AND REMARKS

A concept of the irrotational sound intensity is developed in this work. The irrotational intensity is the gradient of the scalar potential of active intensity. The rotational part is removed. Methods for calculation of the irrotational sound intensity are provided. Examples of the irrotational sound intensity for a normal mode of a plate and for a source composed three monopoles show that the irrotational sound intensity can exactly characterize 'sources' and 'sinks' regions on a complex radiator. With suppression of the field vorticity, a good estimation of the output power can be obtained at some distance without loss in the spatial resolution, as shown in Fig. 3.

REFERENCES


